TRANSITIONAL PROCESSES IN SHEAR FLOWS OF A VISCOELASTIC FLUID. II. INTERACTION OF THE FLUID WITH THE MEASUREMENT SYSTEM

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The nonsteady motion of coaxial cylinders separated by a viscoelastic fluid is investigated.

This article examines the joint nonsteady motion of a viscoelastic fluid and an internal cylinder coupled with an elastic torsion bar. The motion is induced by the impulsive rotation of an external cylinder (problem 2) [1]. The impulsive communication of the motion to the outer cylinder leads to the propagation of a shear wave across the gap in the direction of the internal cylinder. This phenomenon was analyzed in [1]. Two qualitatively different situations may be realized, depending on the relation between the time of passage of the wave across the gap t_W and the relaxation time of the fluid λ. If $\mathrm{El} \ll 1$, then $t_B \gg \lambda$ [1]. In this case, the inner cylinder is moving in an almost purely viscous fluid. The effect of the elastic properties of the fluid is negligible. When $\mathrm{El} \gg 1$, then $t_B \ll \lambda$ [1]. In this situation, the elastic properties of the fluid affect its motion in the gap and the motion of the inner cylinder. The shear wave approaching the inner cylinder leads to its rotation. This in turn causes the propagation of a shear wave in the opposite direction. A quasisteady flow regime is established after several passages of the shear wave across the gap. In this regime, the shear stresses $2\pi r^2 \tau L$ across the gap remain nearly constant, and the inertia of the fluid can be ignored. We will qualitatively analyze the effect of the rheological properties of the fluid on the character of rotation of the inner cylinder at the quasisteady stage for Oldroyd's rheological model, which consists of a Maxwell element and a Newtonian element with the viscosity $\beta\eta_0$, $0 \leq \beta \leq 1$. The parameter β determines the ratio of the contributions of the Newtonian and Maxwell elements to the effective viscosity of the fluid. The motion of the cylinder and the stresses developed in the fluid are described by the following system of equations:

$$I\ddot{\varphi} - 2\pi R_1^2 L(\tau_I + \tau_{II}) + \varkappa\varphi = 0,$$

$$\tau_I + \lambda \frac{d\tau_I}{dt} = \eta_1 (U/h\delta_2 - \dot{\varphi}/\delta_1),\ \ \tau_{II} = \eta_2 (U/h\delta_2 - \dot{\varphi}/\delta_2), \qquad (1)$$

where $\eta_1 = \eta_0(1-\beta)$; $\eta_2 = \eta_0\beta$; $\delta_1 = \dfrac{(2+\delta)\delta}{2(1+\delta)^2} < 1$; $\delta_2 = \delta_1 \dfrac{1+\delta}{\delta} < 1$, and the remaining notation is explained in [1]. These equations were obtained from a system of compatible equations of fluid flow and internal-cylinder motion [1] which ignored the inertia of the fluid. We will write the initial conditions for these equations in the following form

$$\varphi(0) = 0,\ \ \dot{\varphi}(0) = \Omega,\ \ \tau_I(0) = 0. \qquad (2)$$

Such initial conditions allow for consideration of the angular velocity of the inner cylinder Ω acquired while the quasisteady flow regime is being established. It may be assumed that $\Omega \approx 0$ for $I_0 \ll I$. When $I_0 \gg I$, both cylinders acquire the same angular velocity $\Omega = U/R_2$ immediately after passage of the shear wave. After introduction of the quantities:

$$\varphi = \tilde{\varphi} + \varphi_{st},\ \ \tau_I = \tau_{I st} + \tilde{\tau}_I,$$

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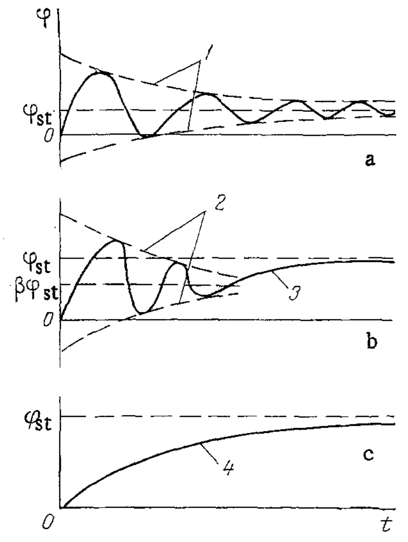

Fig. 1

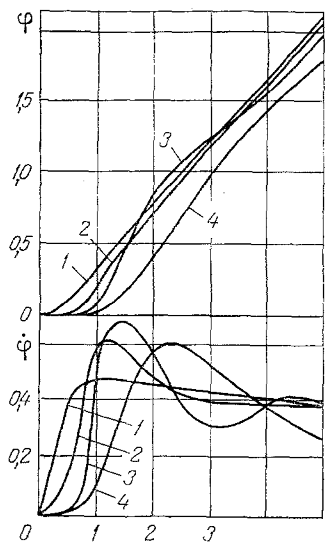

Fig. 2

Fig. 1. Time dependence of angular deflection of internal cylinder in limiting cases of weak (a, b) and strong (c) damping: a) for viscous and viscoelastic ($\lambda\omega_0 \ll 1$) fluids $\xi = 1$; for a viscoelastic fluid ($\lambda\omega_0 \gg 1$, $\varepsilon\lambda \ll 1$) $\xi = \beta$; b) for a viscoelastic fluid ($\lambda\omega_0 \gg 1$, $\varepsilon\lambda \gg 1$); c) for viscous and viscoelastic ($\lambda\omega_0^2/\varepsilon \ll 1$) fluids: $\alpha = 1$; $b = \omega_0^2/2\varepsilon$; for a viscoelastic fluid ($\lambda\varepsilon \gg 1$, $\lambda\omega_0^2/\varepsilon \gg 1$) $\alpha = 1$; $b = \omega_0^2/2\varepsilon$; for a viscoelastic fluid ($\lambda\varepsilon \gg 1$, $\lambda\omega_0^2/\varepsilon \gg 1$) $\alpha = -\beta$, $b = 1/\beta\lambda$ (1) $\varphi_{st} \pm \sqrt{(\varphi_{st})^2 + (\Omega/\omega_0)^2}\exp(-\xi\varepsilon t)$; 2) $\beta\varphi_{st} \pm \sqrt{(\beta\varphi_{st})^2 + (\Omega/\omega_0)^2} \cdot \exp(-\beta\varepsilon t)$; 3) $\varphi \approx \varphi_{st}(1 - (1 - \beta)\exp(-t/\beta\lambda))$; 4) $\varphi \approx \varphi_{st}(1 - be^{-\alpha t})$; the + sign pertains to the top curve, while the − sign pertains to the bottom curve.

Fig. 2. Angular deflection (φ) and angular velocity (φ) of internal cylinder in a viscous (1) and viscoelastic (2-4) fluid during the time $t/\sqrt{I/\varkappa}$; 2) El = 1, $\alpha = 2$; 3) 1 and 4; 4) 10 and 2.

$$\varphi_{st} = \frac{U}{h}\frac{\varepsilon}{\omega_0^2}\frac{\delta_1}{\delta_2}, \quad \tau_{I_{st}} = \eta_1 U/h\delta_2,$$

$$\tau_I = \frac{\eta_1}{\delta_1}\bar{\tau}_I, \quad \omega_0^2 = \varkappa/I, \quad \varepsilon = \frac{\pi R_1^2 L\eta_0}{\delta_1 I}$$

system (1), (2) is converted to the following form, in variables which are displaced relative to the steady state

$$\ddot{\tilde{\varphi}} - 2(1-\beta)\,\varepsilon\bar{\tau}_I + 2\beta\varepsilon\dot{\tilde{\varphi}} + \omega_0^2\tilde{\varphi} = 0,$$

$$\bar{\tau}_I + \lambda\frac{d\bar{\tau}_I}{dt} = -\dot{\tilde{\varphi}}, \tag{3}$$

$$\tilde{\varphi}(0) = -\varphi_{st}, \quad \dot{\tilde{\varphi}}(0) = \Omega, \quad \bar{\tau}_I(0) = -U/h.$$

For a viscous fluid, (3) leads to the familiar equation for the motion of an internal cylinder

$$\ddot{\tilde{\varphi}} + 2\varepsilon\dot{\tilde{\varphi}} + \omega_0^2\tilde{\varphi} = 0, \tag{4}$$

the solutions of which (Fig. 1) are as follows for weak ($\varepsilon/\omega_0 \ll 1$) and strong ($\omega_0/\varepsilon \ll 1$) dampings:

$$\tilde{\varphi} = -\varphi_{st}\cos(\omega_0 t)\exp(-\varepsilon t) + \frac{\Omega}{\omega_0}\sin(\omega_0 t)\exp(-\varepsilon t), \tag{5}$$

TABLE 1. Asymptotic Formulas for the Angular Deflection of the Internal Cylinder during Motion in a Viscoelastic Fluid

No.	Limiting cases		Roots of the characteristic equation	$\tilde{\varphi}=\varphi-\varphi_{st}$
1.1	$\frac{\varepsilon}{\omega_0}\ll 1$	$\lambda\omega_0\ll 1$	$k_1=-1/\lambda+2(1-\beta)\varepsilon$, $k_{2,3}=-\varepsilon\pm i\omega_0[1+(1-\beta)\lambda\varepsilon]$	$-\varphi_{st}\,e^{-\varepsilon t}\cos\{\omega_0[1+(1-\beta)\lambda\varepsilon]t\}+\frac{\Omega}{\omega_0}e^{-\varepsilon t}\sin\{\omega_0[1+(1-\beta)\lambda\varepsilon]t\}$
1.2	$\frac{\varepsilon}{\omega_0}\ll 1$	$\lambda\omega_0\gg 1$	$k_1=-1/\lambda$, $k_{2,3}=-\beta\varepsilon\pm i\omega_0\left(1+\frac{(1-\beta)\varepsilon}{\lambda\omega_0^2}\right)$	$-(1-\beta)\varphi_{st}\,e^{-t/\lambda}-\beta\varphi_{st}\,e^{-\beta\varepsilon t}\cos\left\{\omega_0\left(1+\frac{(1-\beta)\varepsilon}{\lambda\omega_0^2}\right)t\right\}+\frac{\Omega}{\omega_0}e^{-\beta\varepsilon t}\times$ $\times\sin\left\{\omega_0\left(1+\frac{(1-\beta)\varepsilon}{\lambda\omega_0^2}\right)t\right\}$
2.1	$\frac{\omega_0}{\varepsilon}\ll 1$	$\lambda\varepsilon\ll 1$, $\frac{\lambda\omega_0^2}{\varepsilon}\ll 1$,	$k_1=-1/\lambda+2(1-\beta)\varepsilon$, $k_2=-2\varepsilon+4(1-\beta)\varepsilon^2\lambda$, $k_3=-\omega_0^2/2\varepsilon$	$-\varphi_{st}\,e^{-\frac{\omega_0^2}{2\varepsilon}t}+\frac{\Omega}{2\varepsilon}\left(e^{-\frac{\omega_0^2}{2\varepsilon}t}-e^{-2\varepsilon t}\right)$
2.2	$\frac{\omega_0}{\varepsilon}\ll 1$	$\lambda\varepsilon\gg 1$, $\frac{\lambda\omega_0^2}{\varepsilon}\gg 1$	$k_1=-1/\beta\lambda-\frac{(1-\beta)}{2\beta^3\varepsilon\lambda^2}$, $k_2=-2\beta\varepsilon+\left(\frac{1}{\beta}-1\right)\Big/\lambda$, $k_3=-\omega_0^2/2\varepsilon$	$-\varphi_{st}\,e^{-\frac{\omega_0^2}{2\varepsilon}t}+\frac{\Omega}{2\varepsilon}\left(e^{-\frac{\omega_0^2}{2\varepsilon}t}-e^{-t/\lambda\beta}\right)+\frac{\Omega}{2\beta\varepsilon}\left(e^{-t/\lambda\beta}-e^{-2\beta\varepsilon t}\right)$
2.3	$\frac{\omega_0}{\varepsilon}\ll 1$	$\lambda\varepsilon\gg 1$, $\frac{\lambda\omega_0^2}{\varepsilon}\gg 1$	$k_1=-1/\lambda+2(1-\beta)\varepsilon/\omega_0^2\lambda^2$, $k_2=-2\beta\varepsilon+\frac{1}{2\beta}\frac{\omega_0^2}{\varepsilon}$, $k_3=-\omega_0^2/2\beta\varepsilon$	$-\varphi_{st}\,e^{-\frac{\omega_0^2}{2\beta\varepsilon}t}-(1-\beta)\varphi_{st}\left(e^{-\frac{\omega_0^2}{2\beta\varepsilon}t}-e^{-t/\lambda}\right)+\frac{\Omega}{2\beta\varepsilon}\left(e^{-\frac{\omega_0^2}{2\beta\varepsilon}t}-e^{-2\beta\varepsilon t}\right)$
3.1	$\frac{\omega_0-\varepsilon}{\omega_0}\ll 1$	$\varepsilon\lambda\ll 1$, $\omega_0\lambda\ll 1$	$k_1=-1/\lambda+2(1-\beta)\varepsilon$, $k_{2,3}=-\varepsilon-2\varepsilon^2\lambda(1-\beta)\pm$ $\pm i\omega_0\sqrt{2\frac{\omega_0-\varepsilon}{\omega_0}+2(\beta-1)\varepsilon\lambda}$	$-\varphi_{st}\,e^{-\varepsilon t}\cos(A\omega_0 t)-\frac{\varphi_{st}}{A}e^{-\varepsilon t}\sin(A\omega_0 t)+\frac{\Omega}{A\omega_0}e^{-\varepsilon t}\sin(A\omega_0 t)$, $A=\sqrt{2\cdot\frac{\omega_0-\varepsilon}{\omega_0}+2(\beta-1)\varepsilon\lambda}$
3.2	$\frac{\omega_0-\beta\varepsilon}{\omega_0}\ll 1$	$\varepsilon\lambda\gg 1$, $\omega_0\lambda\gg 1$	$k_1=-1/\lambda+2(1-\beta)\frac{\varepsilon}{\omega_0^2\lambda^2}$, $k_{2,3}=-\omega_0-\left(1-\frac{1}{\beta}\right)\frac{1}{\lambda}\pm$ $\pm i\omega_0\sqrt{2\frac{\omega_0-\beta\varepsilon}{\omega_0}+\left(\frac{1}{\beta}-1\right)\frac{2}{\omega_0\lambda}}$	$-(1-\beta)\varphi_{st}\,e^{-t/\lambda}+\beta\varphi_{st}\,e^{-\beta\varepsilon t}\left[\cos(\omega_0 Bt)+\frac{1}{B}\sin(\omega_0 Bt)\right]+$ $+\frac{\Omega}{B\omega_0}e^{-\beta\varepsilon t}\sin(\omega_0 Bt)$, $B=\sqrt{2\frac{\omega_0-\varepsilon}{\omega_0}+\left(\frac{1}{\beta}-1\right)\frac{2}{\omega_0\lambda}}$

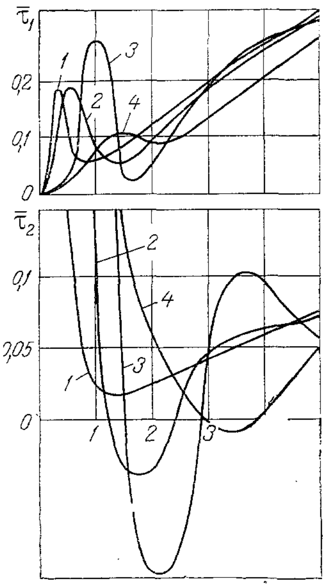

Fig. 3

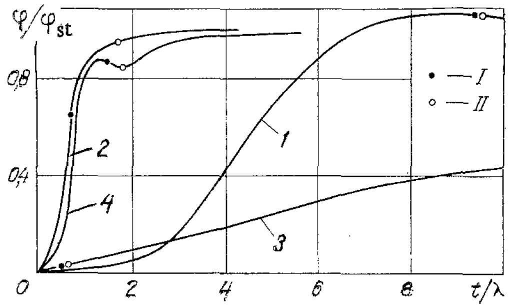

Fig. 4

Fig. 3. Development of shear stresses on internal ($\bar{\tau}_1$) and external ($\bar{\tau}_2$) cylinders for a viscous (1) and viscoelastic (2-4) fluid during the time $t/\sqrt{I/\varkappa}$; 2) El = 1, $\alpha = 2$; 3) 1 and 4; 4) 10 and 2.

Fig. 4. Angular deflection of an internal cylinder in a viscoelastic fluid: 1) El = 0.1; $\varepsilon/\omega_0 = 0.838$; $\omega_0^2\lambda/\varepsilon = 1.193$; $\varepsilon\lambda = 0.838$; $\omega_0\lambda = 1$; 2) 1; 0.0838; 1193; 8.38; 100; 3) 100; 83.8; 0.1193; 837.8; 10; 4) 1; 0.838; 11.93; 8.38; 10; I) $\tau_1/\tau_2 = (1+\delta)^2$; II) $\tau_1/\varphi = \tau_{2st}/$ st.

$$\tilde{\varphi} = -\varphi_{st}\exp\left(-\frac{\omega_0^2}{2\varepsilon}t\right) + \frac{\Omega}{2\varepsilon}\left[\exp\left(-\frac{\omega_0^2}{2\varepsilon}t\right) - \exp(-2\varepsilon t)\right]. \tag{6}$$

When the viscous drag of the fluid and the elasticity of the torsion bar are commensurate ($(\varepsilon - \omega_0)/\omega_0 \ll 1$), the equation for the relative deflection $\tilde{\varphi}$ takes the form:

$$\tilde{\varphi} = -\varphi_{st}\exp(-\varepsilon t)\cos(\omega t) - \frac{\varphi_{st}\omega_0}{\omega}\exp(-\varepsilon t)\sin(\omega t) + \frac{\Omega}{\omega}\exp(-\varepsilon t)\sin(\omega t),\quad \omega = \omega_0\sqrt{2\frac{\omega_0 - \varepsilon}{\omega_0}}. \tag{7}$$

We will subsequently analyze the conditions of motion of the cylinder for two limiting cases: $\Omega = 0\,(I_0 \ll I)$ and $\Omega = U/R_2 = \varphi_{st}\frac{\omega_0^2}{2\varepsilon}(I_0 \gg I)$, corresponding respectively to large and small moments of inertia of the internal cylinder. It can be seen from (5) that in the case of weak damping the inner cylinder completes damped oscillations with a frequency ω_0 and a damping factor ε. Here, the vibration of the cylinder with a low moment of inertia occurs with an amplitude which is $\sqrt{(\omega_0/2\varepsilon)^2 + 1}$ times greater than that of the vibrations of the cylinder with a high moment of inertia. There is also a shift in phase $\varphi_0 = \mathrm{arctg}(\omega_0/2\varepsilon)$. With strong damping, an internal cylinder with a low or high moment of inertia moves in accordance with the law $\tilde{\varphi} = -\varphi_{st}\exp\left(-\frac{\omega_0^2}{2\varepsilon}t\right)$. The solution (6) corresponds to the case when the inertial forces in (4) can be ignored, i.e., when the motion is described by the equation $2\varepsilon\dot{\tilde{\varphi}} + \omega_0^2\tilde{\omega} = 0$. For the case of comparable torsion-bar elasticity and viscous drag ($(\varepsilon - \omega_0)/\omega_0 \ll 1$), the character of the motion of the inner cylinder will again be monotonic, but the motion will occur in the time $1/\varepsilon$. Analysis of the solution of problem (3) in a viscous fluid permits us to distinguish the following characteristic scales: $1/\varepsilon$ — the oscillation damping time for the internal cylinder; ε/ω_0^2 — the time for damping the aperiodic motion of the internal cylinder; $2\pi/\omega_0$ — the period of natural vibration of the cylinder. The possible types of motion of the internal cylinder in a viscoelastic fluid depend on the relation between the relaxation time λ and the time scales for the viscous fluid. Table 1 shows results of analysis of the roots of the following characteristic equation of system (3) for the limiting cases

TABLE 2. Results of Numerical Calculations for Different Numbers of Relaxation Oscillators

N	Viscous fluid	1	2	3	4	5	6	14	24
$\bar{t}_m$	1,145	1,350	1,260	1,220	1,205	1,200	1,200	1,195	1,195
φ	0,396	0,428	0,374	0,351	0,343	0,340	0,340	0,337	0,337
$u_{1\max}$	0,475	0,561	0,602	0,612	0,614	0,614	0,615	0,616	0,616
$\bar{\tau}_1\cdot 10$	0,629	0,677	0,589	0,552	0,541	0,536	0,532	0,531	0,530
$\bar{\tau}_2\cdot 10$	0,189	0,137	0,201	0,279	0,314	0,326	0,323	0,334	0,334

$$k^3+\left(\frac{1}{\lambda}+2\beta\varepsilon\right)k^2+\left(\frac{2\varepsilon}{\lambda}+\omega_0^2\right)k+\omega_0^2/\lambda=0\,.$$

Figure 1 presents asymptotic relations for the angular deflection of the inner cylinder in a viscoelastic fluid. Let us analyze the limiting cases in greater detail.

1. Weak Damping ($\varepsilon/\omega_0\ll 1$). In this case, the natural vibration period of the cylinder is considerably shorter than the time of damping of internal cylinder oscillation in the viscous fluid. The effect of inertial forces is great, and viscous drag is negligible. The cylinder completes damped oscillations, regardless of the relaxation time of the fluid. The character of the damping is determined by the relation between relaxation time λ and the natural vibration period of the cylinder.

1.1. The relaxation time λ is considerably less than the natural period of the cylinder ($\lambda\omega_0\ll 1$). Since $\lambda\dot{\bar{\tau}}_I\sim\lambda\omega_0\bar{\tau}_I\ll\bar{\tau}_I$, then the term $\lambda\dot{\bar{\tau}}_I$ in the rheological equation of system (3) can be ignored. The motion of the inner cylinder will be the same as in the viscous fluid. The effect of the elastic properties of the fluid will lead only to a small increase in the natural frequency of the cylinder compared to the viscous fluid $\omega=\omega_0(1+(1-\beta)\varepsilon\lambda)$.

1.2. The relaxation time λ is considerably greater than the natural period of the cylinder ($\lambda\omega_0\gg 1$). The elastic forces of the fluid have a substantial effect on the motion of the cylinder. There is a qualitative change in the character of motion of the cylinder, depending on the relation between relaxation time λ and the damping scale $1/\varepsilon$.

1.2a. The relaxation time is considerably less than the damping time $1/\varepsilon$ ($\lambda\varepsilon\ll 1$). The change in shear stress succeeds in "tuning in" to the change in cylinder vibration amplitude. The inertial forces and the elastic forces of the torsion bar are substantial, and viscous drag is negligible. Here $\lambda\dot{\bar{\tau}}_I\sim\lambda\omega_0\dot{\bar{\tau}}_I\gg\bar{\tau}_I$, and the rheological equation yields $\bar{\tau}_I\sim-\tilde{\varphi}/\lambda$. The equation of motion of the internal cylinder takes the form

$$\ddot{\tilde{\varphi}}+2\beta\varepsilon\dot{\tilde{\varphi}}+\left(\omega_0^2+\frac{2\varepsilon(1-\beta)}{\lambda}\right)\tilde{\varphi}=0,$$

from which it follows that the Maxwell element of the flow equation acts as an elastic element, increasing the natural frequency of the cylinder $\omega_0=\omega_0(1+\varepsilon(1-\beta)/\lambda\omega_0^2)$, and the damping factor decreases to the value $\beta\varepsilon$. The behavior of the stresses in the Maxwell element is similar to the change in $\tilde{\varphi}$. Since $\bar{\tau}=\bar{\tau}_I+\bar{\tau}_{II}=\bar{\tau}_I-\beta\dot{\tilde{\varphi}}\sim-\tilde{\varphi}/\lambda-\beta\omega_0\tilde{\varphi}$, and $\omega_0\lambda\gg 1$, then $\bar{\tau}\sim-\beta\omega_0\tilde{\varphi}$ as well. System (3) can be solved for the case being considered by solving the motion equation of the cylinder in the viscous fluid with the substitution of $\beta\varepsilon$ for ε and $\omega_0(1+\varepsilon(1-\beta)/\lambda\omega_0^2)$ for ω_0. The cylinder oscillation damping time in the viscoelastic fluid is $1/\beta$ times greater than in the viscous fluid.

1.2b. The relaxation time is significantly greater than the characteristic damping scale ($\varepsilon\lambda\gg 1$). The stress of the Maxwell element changes very slowly. The derivatives $\dot{\tilde{\varphi}}$ and $\ddot{\tilde{\varphi}}$ in the motion equation of the inner cylinder can be ignored, and $\bar{\tau}_I$ is determined from the expression $\bar{\tau}_I\approx\omega_0^2\tilde{\varphi}/2\varepsilon$. Substituting $\bar{\tau}_I$ into the flow equation with allowance for the fact that $\omega_0^2\lambda/2\varepsilon\gg 1$, we obtain $\tilde{\varphi}\sim\exp(-t/\lambda)$. Since the total stress $\bar{\tau}\sim\tilde{\varphi}$, then the character of motion of the inner cylinder in the case being examined is determined by the change in $\bar{\tau}$. The motion is oscillatory in nature for $t\lesssim 1/\beta\varepsilon$. For $t\gtrsim 1/\beta\varepsilon$, the angular deflection increases monotonically: $\varphi\approx\varphi_{st}(1-(1-\beta)\exp(-t/\lambda))$. For a cylinder with a high moment of inertia, φ changes without exceeding φ_{st} when $\beta<1/2$. The range of the oscillations φ is less $2\beta\varphi_{st}<\varphi_{st}$ up to the time $t\gtrsim 1/\beta\varepsilon$, while φ monotonically increases to φ_{st} for $t\gtrsim 1/\beta\varepsilon$. For $\beta>1/2$, the oscillations take place up to $t\sim 1/\beta\varepsilon$ and occur with an increase in φ_{st}. The value of φ also monotonically approaches φ_{st} for $t \gg 1/\beta_e$. For a cylinder with a low moment of inertia, the character of the dependence of φ on time does not change qualitatively, but the oscillations occur with a considerably greater amplitude.

2. Strong Damping ($\omega_0/\varepsilon \ll 1$). The motion of the cylinder is always damping and nonoscillatory, regardless of the relaxation properties of the fluid. It is characterized by two times: $1/\varepsilon$ and ε/ω_0^2. The effect of the torsion bar is negligible in the interval $(0, 1/\varepsilon)$. Only the inertial forces and drag forces are significant. The effect of the inertial forces can be ignored in the interval $(1/\varepsilon, \varepsilon/\omega_0^2)$. Different regimes of cylinder motion are possible, depending on the relation between relaxation time λ and the characteristic times.

2.1. Relaxation time λ is significantly less than the characteristic damping time $1/\varepsilon$ ($\lambda\varepsilon \ll 1$). The condition $\omega_0^2\lambda/\varepsilon \ll 1$ is also satisfied in this case. This means that the stiffness of the fluid layer is significantly greater than the stiffness of the torsion bar. The elastic properties of the fluid do not affect the motion of the cylinder. As in the viscous fluid, cylinder motion occurs in this case with strong damping. The formula for φ is obtained by ignoring inertial forces in the cylinder motion equation and the time derivative of stress in the flow law.

With $\varepsilon\lambda \gg 1$, when the relaxation time λ significantly exceeds the characteristic scale $1/\varepsilon$, the elastic properties of the fluid affect the character of motion of the cylinder in the first interval $(0, 1/\varepsilon)$. Their effect in the second interval $(1/\varepsilon, \varepsilon/\omega_0^2)$ depends on the relation between the relaxation time of the fluid λ and the characteristic time ε/ω_0^2.

2.2. The relaxation time ε/ω_0^2 is considerably less than the characteristic time $(\omega_0^2\lambda/\varepsilon \ll 1)$. It is located within the interval $(1/\varepsilon, \varepsilon/\omega_0^2)$. Here, the stiffness of the torsion bar is considerably less than the stiffness of the fluid layer. For $t \lesssim 1/\varepsilon$, $\varphi \approx \Omega(1 - \exp(-2\beta\varepsilon t))/2\beta\varepsilon$. The elastic properties of the fluid affect the motion of the inner cylinder up to the time $t \lessgtr \beta\lambda$. At $t \gtrsim \beta\lambda$, the elasticity of the fluid does not effect the angular deflection φ, which changes in the same manner as in the viscous fluid: $\tilde{\varphi} \sim \exp(-(\omega_0^2/2\varepsilon)t)$. The behavior of φ is nearly the same in both limiting cases $\Omega = 0$ and $\Omega = \omega_0^2\varphi_{st}/2\varepsilon$.

2.3. The relaxation time λ is significantly greater than the characteristic times $1/\varepsilon$, ε/ω_0^2. Here the stiffness of the torsion bar is considerably greater than the stiffness of the fluid layer $(\omega_0^2\lambda/\varepsilon \gg 1)$. The elastic properties of the fluid determine the motion of the cylinder until φ reaches its steady-state value. As in the preceding case for $t \lesssim 1/\varepsilon$, the rotation of the internal cylinder occurs in accordance with the law $\varphi \approx \Omega(1 - \exp(-2\beta\varepsilon t))/2\beta\varepsilon$. The cylinder deflects through the angle $\beta\varphi_{st}$ over a period of time of the order $2\beta\varepsilon/\omega_0^2$, after which the deflection angle increases monotonically: $\varphi \approx \varphi_{st}(1 - (1 - \beta)\exp(-t/\lambda))$. In fact, since the inner cylinder moves slowly in this case, the derivatives $\dot{\tilde{\varphi}}$ and $\ddot{\tilde{\varphi}}$ in the motion equation can be ignored. Then the stresses in the Maxwell element change in a manner similar to the angular deflection: $\bar{\tau}_I \approx \frac{\omega_0^2}{2\varepsilon(1-\beta)}\tilde{\varphi}$, and it follows from the flow law that $\tilde{\varphi} \sim \exp(-t/\lambda)$. The solution obtained shows that the value of the moment of inertia of the cylinder does not change the monotonic nature of the increase in φ.

3. The elastic properties of the torsion bar are comparable to the viscous drag forces of the fluid ($\varepsilon \sim \omega_0$). The angular deflection increases monotonically, regardless of the relaxation time λ and the moment of inertia of the cylinder. In the case

3.1. the elasticity of the fluid has almost no effect on the character of motion of the cylinder. In contrast to the corresponding case for a viscous fluid, the steady-state value φ_{st} is established more slowly. In the case 3.2 for $t \lesssim 1/\beta\varepsilon$, $\varphi \sim \varphi_{st}\beta(1 - \exp(-\beta\varepsilon t))$. The cylinder deflects through the angle $\beta\varphi_{st}$ in the time interval $t \sim 1/\beta\varepsilon$, with the angle then increasing monotonically: $\varphi \approx \varphi_{st}(1 - (1 - \beta)\exp(-t/\lambda))$. The effect of the elastic properties of the fluid is manifest for the entire time of motion of the cylinder.

The completed analysis shows that the manifestation of elastic properties of the fluid depends heavily on the relations between its rheological parameters and the characteristics of the mechanical system. This is most clearly seen in cases 1.2, 2.3, and 3.2 (Fig. 1), when stress relaxation is the longest process.

Let us analyze the results of numerical investigations of the problem in question for a Maxwell liquid with a relaxation time spectrum. The calculations were performed for the same cylinder. The viscosity and relaxation properties of the fluid were varied. The mathematical formulation of the problem was given in [1]. The development of the shear wave was also analyzed in detail in [1]. We will examine the transitional stage, which begins after the shear wave approaches the inner cylinder and continues until steady-state fluid flow is established. The motion of the cylinder in this stage is qualitatively similar for all of the variants calculated. We will give a more detailed account of the results for the case when the character of the damping is characterized by the condition $\varepsilon/\omega_0 = 8.378$. Here an increase in $El(\omega_0\lambda)$ from 1 to 10 leads to an increase in $\omega_0^2\lambda/\varepsilon$ from 0.1193 to 1.193 and in $\varepsilon\lambda$ from 8.378 to 83.78.

Receiving an impulse from the shear wave, the internal cylinder begins to rotate with increasing velocity. With an increase in El and α, the moment of time $\bar{t}_m$ at which the velocity of the cylinder reaches its maximum value increases. Thus, if $\bar{t}_m = 1.145$ for a viscous fluid, then $\bar{t}_m$ will increase from 1.195 to 1.380 with a change in α from 2 to 4 with El = 1. With an increase in El to 10 for $\alpha = 2$, we have $\bar{t}_m = 2.300$. The maximum value of the speed of rotation of the cylinder $\bar{u}_{1max}$ characterizes the magnitude of the impulse the cylinder receives from the fluid. The numerical calculations and the analytical results in [1] showed that the value of $\bar{u}_{1max}$ depends on the parameter α and changes slightly with a change in El: at $\alpha = 2$, $\bar{u}_{1max} = 0.6157$ for El = 1 and 0.5997 for El = 10. At the same time, with $\alpha = 4$ and El = 1, $\bar{u}_{1max} = 0.8422$. For a viscous fluid (El = 0), $\bar{u}_{1max} = 0.475$.

The angle of deflection of the internal cylinder from the equilibrium position φ increases monotonically for both viscous and viscoelastic fluids (Fig. 2), reaching a steady-state value $\varphi_{st} = 8.378$ for the case in question at $t \to \infty$. As already noted, the presence of a fluid with elastic properties leads to a reduction in the velocity of shear disturbances. Thus, for $t \ll t_w$, when the shear wave has not yet reached the internal cylinder, the values of φ and $\dot{\varphi}$ are determined mainly only by the presence of the viscous terms in the flow equation [1], for which $\lambda_k t_w \ll 1$, and these values are very small compared to the corresponding situation for a viscous fluid (Fig. 2).

For El $\gg$ 1, the motion of the cylinder after the elastic shear wave reaches its surface is determined to a substantial degree by the elastic properties of the fluid. An increase in the parameter α decreases the effective viscosity at $t \ll \lambda$ [1]. This leads to an increase in the velocity of the internal cylinder and a corresponding increase in the angle φ (curve 3 in Fig. 2). The development of the shear stresses on the inner and outer cylinders is shown in Fig. 3. In a viscous fluid, the curve $\bar{\tau}_2(\bar{t})$ has a characteristic minimum $\bar{\tau}_{2min}$ in the region of positive values. There is then a monotonic increase in $\bar{\tau}_2$. The presence of a fluid with elastic properties shifts $\bar{\tau}_{2min}$ into the region of negative values. A further increase in $\bar{\tau}_2(\bar{t})$ for a fluid having elasticity is accompanied by an oscillatory change next to the corresponding values for the viscous fluid. An increase in α reinforces the oscillatory character of the development of $\bar{\tau}_2$, while an increase in El has the opposite effect. Also, with an increase in both El and α, the moment of onset of $\bar{\tau}_{2min}$ is shifted into the region of large times. This is connected with a reduction in the rate of development of the shear perturbations.

The presence of a fluid with relaxation properties and, foremost, time-dependent effective viscosity, leads to a situation whereby the curves of shear stress development on the internal cylinder $\bar{\tau}_1 = \bar{\tau}_1(\bar{t})$ for a viscoelastic fluid always lie below the corresponding curves for a viscous fluid in the initial stage of flow. The shear stresses develop on a nearly stationary cylinder during this stage. The effect of the elastic forces of the torsion bar can be ignored during this period. The subsequent development of shear stresses on the internal cylinder is qualitatively similar for viscous and viscoelastic fluids: The curves $\bar{\tau}_1 = \bar{\tau}_1(\bar{t})$ have characteristic regions of a local maximum and minimum in the transitional stage of flow. Here, for the viscoelastic fluid, a reduction in the velocity of the shear wave with an increase in α and El shifts the extremums of $\bar{\tau}_1$ in the direction of higher times. The determining role at this stage is played by two opposing factors: first, the increase in shear stress on the internal cylinder as a stationary cylinder, which depends on the properties of the fluid and the shear velocity communicated to the cylinder by the fluid; second, the shear stress due to the propagation of a shear wave from the internal cylinder after it is brought into motion and having a value proportional to the angular velocity of the cylinder $\dot{\varphi}(t_w)$.

Since the angular velocity of the cylinder $\dot{\varphi}(t_w)$ during the initial period is slightly dependent on the value of the parameter El, then, allowing for the decrease in dynamic viscosity, an increase in El will be accompanied by smoother development of the shear stresses on the internal cylinder. An increase in the parameter α will be accompanied by an increase in angular velocity $\dot{\varphi}(t_W)$ and the "ranges" of the oscillatory change of $\bar{\tau}_1$. The effect of the number of terms in the flow equation [1] on the results of numerical calculation of the dynamics of the transitional process is shown in Table 2. For comparison, this table gives the kinematic and dynamic characteristics of the transitional process in the flow of a viscoelastic fluid ($El = 1$, $\alpha = 2$) for moments of time corresponding to the maximum velocity of the internal cylinder. It is apparent from Table 2 that the results of the calculations nearly coincide for $N > 6$.

The transitional regime changes to a quasisteady regime with time: the moment of the shear stresses $2\pi r^2 \tau L$ remains nearly constant across the gap, and the ratio of the shear stresses on the inner and outer cylinders is close to the corresponding ratio for steady flow: $\tau_1(t)/\tau_2(t) \approx (1+\delta)^2$. As already noted, the relationship between the rheological constants of the fluid and the parameters of the mechanical system are manifest particularly clearly during the concluding, quasisteady stage of motion. Figure 4 presents results of calculations of a problem on the motion of an internal cylinder in a viscoelastic fluid in its complete formulation [1] (the equations being in partial derivatives). The parameters of the mechanical system were fixed. The viscosity and relaxation time of the fluid were varied. The completed calculations confirmed the feasibility of using the quasisteady approach to analyze the motion of an internal cylinder. Figure 4 indicates the times at which this becomes applicable. However, it must be used with great care. Not all of the sections of the curves for the quasisteady approximation depicted in Fig. 1 are actually realized. Only the sections for $t \gg t_W$ exist. For example, the results of numerical calculations with $\varepsilon/\omega_0 = 0.0838$, $El = 1$, $\omega_0^2\lambda/\varepsilon = 1193$, $\varepsilon\lambda = 8.378$, and $\omega_0\lambda = 100$, corresponding to case 1.2 in Table 1, show that the section of damped oscillations depicted in Fig. 1b at $t \lesssim 1/\beta\varepsilon$ is absent (see Fig. 4). This is due to the fact that

$$\frac{1}{\varepsilon t_{\text{в}}} \sim \left(\frac{Z(\alpha)\alpha\sin\dfrac{\pi}{\alpha}}{\pi\,El}\right)^{\frac{-\alpha}{2\alpha-1}} \frac{[(1+\delta)^4-1]\,I\delta_1}{2\delta^2 I_0\,El} = \left(\frac{\pi}{3}\right)^{-\frac{2}{3}} \frac{3}{8\pi} < 1$$

and this interval lies in the transitional region, where the quasisteady approximation is inapplicable. The circles in Fig. 4 denote the times after which the inertia of the internal cylinder can be ignored and the shear stresses in the fluid can be measured from the angular deflection on the section of nonsteady motion. These results agree with the qualitative analysis of the equation of quasisteady motion of the internal cylinder.

LITERATURE CITED

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